<u>Problem 1</u> *(2.5 points)*

In a nonmagnetic, lossy dielectric medium, a 400 MHz plane wave is characterized by the magnetic field phasor:

$$\tilde{\mathbf{H}} = (\hat{\mathbf{x}} - j4\hat{\mathbf{z}})\, e^{-0.05y} e^{-j10y}$$

Obtain <u>time-domain</u> expressions for: the electric and magnetic field vectors.

a) the electric field vector (1 points)
b) the magnetic field vector (1 points)
c) Determine the polarization of the wave (0.5 point)

(Free space permittivity: ε_0=8.85×10^{-12} F/m, free space permeability: μ_0=4π×10^{-7} H/m)

<u>Solution</u>:

From the expression of the magnetic field phasor, the attenuation and phase constants are known:

$$\alpha = 0.05 \text{ Np/m}$$
$$\beta = 10 \text{ rad/m}$$

Frequency f=500 MHz=5×10^8 Hz and the propagation direction is along y axis. So:

$$-\omega^2 \mu\varepsilon' = -\omega^2 \mu_0 \varepsilon_0 \varepsilon_r' = \alpha^2 - \beta^2 \approx -100$$

Thus, the real part of the relative permittivity is:

$$\varepsilon_r' = \frac{100}{\omega^2 \mu_0 \varepsilon_0} = \frac{100}{(6.28 \times 4 \times 10^8)^2 \times 4\pi \times 10^{-7} \times 8.85 \times 10^{-12}} = 1.426$$

And for the imaginary part of the relative permittivity:

$$2\alpha\beta = \omega^2 \mu\varepsilon'' = \omega^2 \mu_0 \varepsilon_0 \varepsilon_r''$$

So:

$$\varepsilon_r'' = \frac{2\alpha\beta}{\omega^2 \mu_0 \varepsilon_0} = \frac{2 \times 0.05 \times 10}{(6.28 \times 4 \times 10^8)^2 \times 4\pi \times 10^{-7} \times 8.85 \times 10^{-12}} = 0.014$$

Since $\varepsilon_r''/\varepsilon_r' = 0.0098 \ll 1$, the medium has very low loss so we can use low loss approximation.

The intrinsic impedance is:

$$\eta = \sqrt{\frac{\mu}{\varepsilon'}} = \sqrt{\frac{4\pi \times 10^{-7}}{1.426 \times 8.85 \times 10^{-12}}} = 315.5 \ \Omega$$

Electrical field phasor is thus

$$\tilde{\mathbf{E}} = -\eta \hat{\mathbf{k}} \times \tilde{\mathbf{H}} = -315.5 \hat{\mathbf{y}} \times (\hat{\mathbf{x}} - j4\hat{\mathbf{z}})\, e^{-0.05y} e^{-j10y}$$
$$= (j4\hat{\mathbf{x}} + \hat{\mathbf{z}}) 315.5 e^{-0.05y} e^{-j10y}$$

In time domain:

$$\mathbf{E}(t) = \mathrm{Re}\left\{\tilde{\mathbf{E}}e^{j\omega t}\right\}$$
$$= \hat{\mathbf{x}}1262e^{-0.05y}\cos(\omega t - 10y + 90°) + \hat{\mathbf{z}}\,315.5e^{-0.05y}\cos(\omega t - 10y)$$
$$= -\hat{\mathbf{x}}1262e^{-0.05y}\sin(\omega t - 10y) + \hat{\mathbf{z}}\,315.5e^{-0.05y}\cos(\omega t - 10y)$$
$$\mathbf{H}(t) = \mathrm{Re}\left\{\tilde{\mathbf{H}}e^{j\omega t}\right\}$$
$$= \hat{\mathbf{x}}\,e^{-0.05y}\cos(\omega t - 10y + 90°) + \hat{\mathbf{z}}\,4e^{-0.05y}\sin(\omega t - 10y)$$

The polarization of the wave is right-hand elliptical.

Problem 2 *(1.5 points)*

A lossless 100 Ω transmission line of 3λ/8 in length is terminated in an unknown load impedance. The input impedance Z_{in} is measured to be 60+*j*80, determine the load impedance.

Solution:

Input impedance is:

$$Z_{in} = Z_0\frac{z_L + j\tan\beta l}{1 + jz_L\tan\beta l} = 100\frac{z_L + j\tan(3\pi/4)}{1 + jz_L\tan(3\pi/4)} = 100\frac{z_L - j}{1 - jz_L} = 60 + j80$$

Solve for z_L and find $z_L=3$, so the load impedance is 300 Ohm.